Influence of Generalized and Extended Uncertainty Principle on Thermodynamics of Friedmann-Robertson-Walker universe

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The influence of the generalized uncertainty principle (GUP) and extended uncertainty principle (EUP) on the thermodynamics of the Friedmann-Robertson-Walker (FRW) universe has been investigated. It is shown that the entropy of the apparent horizon of the FRW universe gets a correction if one considers the effect of the GUP or EUP. Moreover, starting with the modified entropy and applying the first law of thermodynamics, $dE = TdS$, to the apparent horizon of the FRW universe, we obtain the modified Friedmann equations. The influence of the GUP or EUP on the thermodynamics of the FRW universe provides a deep insight into the understanding of the quantum gravity or large length scale corrections to the dynamics of the FRW universe.

I. INTRODUCTION

The four thermodynamics laws of black hole, which were originally derived from the classical Einstein Equation, provide deep insight into the connection between thermodynamics and Einstein Equation[1, 2]. Recently, this connection has been investigated extensively in the literatures for Rindler spacetime and Friedmann-Robertson-Walker (FRW) universe. For Rindler spacetime[3], the Einstein equation can be derived from the proportionality of entropy to the horizon area, together with the Clausius relation $\delta Q = TdS$. Here δQ and T are the energy flux and Unruh temperature detected by an accelerated observer just inside the local Rindler causal horizons through spacetime point. In FRW universe[4], after replacing the event horizon of black hole by the apparent horizon of FRW space-time and assuming that the apparent horizon has an associated entropy S and temperature T

$$S = \frac{A}{4G}, \qquad T = \frac{1}{2\pi \tilde{r}_A}, \qquad (1)$$

one can cast the first law of thermodynamics, $dE = TdS$, to the Friedmann equations. Here G, A, and $\tilde{r}_A$ are the gravitational constant, the area of the apparent horizon, and the radius of the apparent horizon, respectively. The first law of thermodynamics not only holds in Einstein gravity, but also in Guass-Bonnet gravity, Lovelock gravity, and various braneworld scenarios[5, 6, 7]. The fact that the first law of thermodynamics holds extensively in various spacetime and gravity theories suggests a deep connection between gravity and thermodynamics. (Some other viewpoints and further developments in this direction see [8, 9, 10, 11, 12, 13] and references therein.)

The thermodynamics behavour of spacetime is only one of the features of Einstein gravity. Another feature is the Hawking radiations at the event horizon of black holes[14] or the apparent horizon of the FRW spacetime[15]. The Hawking radiation is a quantum mechanics effect in the classical background black hole or FRW spacetime. Therefore quantum theory, gravitational theory and thermodynamics meet together at black holes and FRW spacetime. For a black hole, it radiates and becomes smaller and hotter, finally disappears when the Hawking radiation ends, leaving behind thermal radiation described by quantum mechanical mixed states.

However, the analysis of Hawking radiation in the literatures usually make use of the semi-classical approaches, assuming a classical background metric and considering a quantum radiation process. When it comes into the high energy regime, for example a small black hole whose size can compare with Planck scale or a FRW universe in the era of Planck time, the effect of quantum gravity should not be forgotten. In these cases, the conventional semi-classical approaches are not proper and the complete quantum theory of gravity is required.

Recently, a growing interest has been focused on the proposal that the quantum gravity effect might need us turn from the usual commutation relations of the Heisenberg's uncertainty principle (HUP) to the generalized uncertainty principle (GUP)[16]. The GUP is a model independent aspect of quantum gravity and can be derived from different approaches to quantum gravity, such as string theory[18], loop quantum gravity and noncommutative quantum mechanics[17].

Naturally, one may think that the GUP should influence the thermodynamics of black holes and FRW universe in the small scale or in the high energy regime. Indeed, this issue has been investigated in contexts of black hole physics. As we have known, the GUP affects the thermodynamics of black holes in two aspects. First, the GUP might modify the Hawking temperature on the event horizon and may prevent the total evaporation of a black hole[19, 20]. Second, after considering the GUP, one will get a correction to the Bekenstein-Hawking en-

tropy of a black hole[21, 22]. This correction modifies the famous entropy-area relation that the entropy of a black hole is proportional to its area of the event horizon. The impact of the GUP on other physics systems has also been investigated extensively, see[27] and references therein.

However, as far as we know, whether the GUP can influence the thermodynamics of FRW universe is still unknown. Is there indeed a correction to the entropy on the apparent horizon of the FRW universe when we consider the effect of the GUP? If the GUP is considered,can we still get the Friedmann equations when we apply the first law of thermodynamics to the apparent horizon? These problems need to be solved. In this paper, we are going to investigate these problems. We find that by utilizing the GUP, the entropy of the apparent horizon of the FRW universe should get a correction. Moreover, starting with the modified entropy on the apparent horizon, we will show that the first law of thermodynamics on the apparent horizon can produce the corresponding modified Friedmann equations.

However, as a high energy correction to HUP, the GUP should not be important for the late time FRW universe. In this case, one might consider the effect of the large length scale modification. (For example, in Dvali-Gabadadze-Porrati braneworld model, the large length scale modification to the Einstein gravity on the brane might lead to the late time acceleration of our universe.) Recently, an extended uncertainty principle (EUP) has been introduced to incorporate the effect of the large length scale[20, 23]. We note here that in this paper, we adopt the terminology, the extended uncertainty principle (EUP) and the generalized EUP (GEUP), which were first used in [20]. Contrary to the GUP, the EUP is the large length scale correction to the Heisenberg's uncertainty principle. In black hole physics, the uncertainty principle can be used to derive the Hawking temperature of Schwarzschild-(anti)de Sitter black hole[20, 24]. Like in the case of the GUP, it is also of interest to investigate the impact of the EUP on the thermodynamics of the FRW universe. In this paper, we will also consider this issue.

Therefore, the organization of this paper is as follows. In Section II, we investigate the influence of the GUP on the thermodynamics of the FRW universe, and in Section III, we generalize the discussions of the GUP to the EUP case. The Section IV are our summary and discussions. Throughout the paper, the units $c \equiv \hbar \equiv k_B \equiv 1$ are used.

II.   THE GUP CASE

Let us begin with the GUP, which is usually given by[16]

$$\delta x \delta p \geq 1 + \alpha^2 l_p^2 \delta p^2, \qquad (2)$$

where l_p is the Planck length, and α is a dimensionless real constant. The GUP has an immediate consequence that there is a minimal length with the Planck scale,

$$\delta x \geq \frac{1}{\delta p} + \alpha^2 l_p^2 \delta p \geq 2|\alpha l_p|. \qquad (3)$$

This minimal length characterizes the absolute minimum in the position uncertainty.

Another consequence of GUP is the modified momentum uncertainty. After some simple manipulations, the momentum uncertainty can be written as

$$\delta p \geq \frac{1}{\delta x}[\frac{\delta x^2}{2\alpha^2 l_p^2} - \frac{\delta x^2}{2\alpha^2 l_p^2}\sqrt{1 - \frac{4\alpha^2 l_p^2}{(\delta x)^2}}] = \frac{1}{\delta x}f_G(\delta x^2), \quad (4)$$

where

$$f_G(\delta x^2) = \frac{\delta x^2}{2\alpha^2 l_p^2} - \frac{\delta x^2}{2\alpha^2 l_p^2}\sqrt{1 - \frac{4\alpha^2 l_p^2}{(\delta x)^2}} \qquad (5)$$

characterizes the departure of the GUP from the Heisenberg uncertainty principle $\delta p \geq 1/\delta x$.

We consider a $(n + 1)$-dimensional FRW universe, whose linear element is given by

$$ds^2 = -dt^2 + a^2(\frac{dr^2}{1 - kr^2} + r^2 d\Omega_{n-1}^2), \qquad (6)$$

where $d\Omega_{n-1}^2$ denotes the line element of an $(n - 1)$-dimensional unit sphere, a is the scale factor of our universe and k is the spatial curvature constant. In FRW spacetime, there is a dynamical apparent horizon, which is a marginally trapped surface with vanishing expansion. Using the notion $\tilde{r} = ar$, the radius of the apparent horizon can be written as

$$\tilde{r}_A = \frac{1}{\sqrt{H^2 + k/a^2}}, \qquad (7)$$

where H is the Hubble parameter, $H \equiv \dot{a}/a$ (the dot represents derivative with respect to the cosmic time t). On the apparent horizon, if we suppose that the apparent horizon has an associated entropy S and temperature T

$$S = \frac{A}{4G}, \qquad T = \frac{1}{2\pi\tilde{r}_A}, \qquad (8)$$

(where A is the apparent horizon area $A = n\Omega_n\tilde{r}_A^{n-1}$ with $\Omega_n = \pi^{n/2}/\Gamma(n/2 + 1)$ being the volume of an n-dimensional unit sphere.) it has been confirmed [4] that the first law of thermodynamics,

$$dE = TdS, \qquad (9)$$

can reproduce the Friedmann equations

$$\dot{H} - \frac{k}{a^2} = -\frac{8\pi G}{n - 1}(\rho + p), \qquad (10)$$

$$H^2 + \frac{k}{a^2} = \frac{16\pi G}{n(n - 1)}\rho. \qquad (11)$$

Here ρ is the energy density of cosmic fluid and $dE = d(\rho V)$ is the energy flow pass through the apparent horizon. Note that in order to get Eq.(11), one should use

$$\dot{\rho} + nH(\rho + p) = 0, \qquad (12)$$

which is the continuity (conservation) equation of the perfect fluid.

Now we consider the impact of the GUP on thermodynamics of FRW universe. We consider the case that the apparent horizon having absorbed or radiated a particle with energy dE. As point out in[20], one can identify the energy of the absorbed or radiated particle as the uncertainty of momentum,

$$dE \simeq \delta p. \qquad (13)$$

By considering the quantum effect of the absorbed or radiated particle, which implies the Heisenberg uncertainty principle $\delta p \geq \hbar/\delta x$, the increase or decrease in the area of the apparent horizon can be expressed as

$$dA = \frac{4G}{T}dE \simeq \frac{4G}{T}\frac{1}{\delta x}. \qquad (14)$$

In the above we didn't consider the impact of the GUP. When the effect of the GUP (4) is considered, the change of the apparent horizon area can be modified as

$$dA_G = \frac{4G}{T}dE \simeq \frac{4G}{T}\frac{1}{\delta x}f_G(\delta x^2). \qquad (15)$$

Using Eq.(14), we have

$$dA_G = f_G(\delta x^2)dA. \qquad (16)$$

Take into account that the position uncertainty δx of the absorbed or radiated particle can be chosen as its Compton length, which has the order of the inverse of the Hawking temperature, one can take[21]

$$\delta x \simeq 2\tilde{r}_A = 2(\frac{A}{n\Omega_n})^{\frac{1}{n-1}}. \qquad (17)$$

Thus, the departure function $f_G(\delta x^2)$ can be re-expressed in terms of A,

$$f_G(A) = \frac{2}{\alpha^2 l_p^2}(\frac{A}{n\Omega_n})^{\frac{2}{n-1}}(1 - \sqrt{1 - \alpha^2 l_p^2(\frac{n\Omega_n}{A})^{\frac{2}{n-1}}}). \quad (18)$$

Here and hereafter we use $f_G(A)$ represent the departure function $f_G(\delta x^2)$. At $\alpha = 0$, we express $f_G(A)$ by Taylor series

$$\begin{aligned} f_G(A) &= 1 + \frac{\alpha^2 l_p^2}{4}(\frac{n\Omega_n}{A})^{\frac{2}{n-1}} + \frac{(\alpha^2 l_p^2)^2}{8}(\frac{n\Omega_n}{A})^{\frac{4}{n-1}} \\ &+ \sum_{d=3} c_d(\alpha l_p)^{2d}(\frac{n\Omega_n}{A})^{\frac{2d}{n-1}}, \end{aligned} \qquad (19)$$

where c_d is a constant.

If we substitute (19) into Eq.(16) and integrating, we can get the modified area A_G from the GUP. Then we can also get the correction to the entropy area relation by using $S_G = A_G/4G$. But integrating Eq.(16) might be complicated and dimensional dependent. Therefore, we should divide our discussions into three cases: $(1)n = 3$; $(2)n > 3$ and n is a even number; $(3)n > 3$ and n is a odd number.

A. The $n = 3$ case

When $n=3$, we have

$$\begin{aligned} f_G(A) &= 1 + \pi\alpha^2 l_p^2 \frac{1}{A} + 2(\pi\alpha^2 l_p^2)^2 \frac{1}{A^2} \\ &+ \sum_{d=3} c_d(4\pi\alpha^2 l_p^2)^{2d}\frac{1}{A^d}. \end{aligned} \qquad (20)$$

Substituting (20) into (16) and integrating, we obtain

$$\begin{aligned} A_G &= A + \pi\alpha^2 l_p^2 lnA - 2(\pi\alpha^2 l_p^2)^2\frac{1}{A} \\ &- \sum_{d=3} \frac{c_d(4\pi\alpha^2 l_p^2)^{2d}}{d-1}\frac{1}{A^{d-1}} + c \end{aligned} \qquad (21)$$

where c is the integral constant. By making use of Bekenstein-Hawking area law, $S = A/4G$, we can obtain the expression of the entropy of the apparent horizon including the effect of the GUP. That is, the modified entropy is given by

$$\begin{aligned} S_G &= \frac{A}{4G} + \frac{\pi\alpha^2 l_p^2}{4G}ln\frac{A}{4G} - 2(\frac{\pi\alpha^2 l_p^2}{4G})^2(\frac{A}{4G})^{-1} \\ &- \sum_{d=3} \frac{c_d(\frac{16\pi^2\alpha^4 l_p^4}{4G})^d}{d-1}(\frac{A}{4G})^{1-d} + const. \end{aligned} \qquad (22)$$

This relation has the standard form of the entropy-area relation as given by other approaches in black holes[25, 26, 28]. The point which should be stressed here is that the coefficient of the logarithmic correction term is positive. This is different with the results in Refs.[25, 26]. As pointed out in some literatures[26], the coefficient of the logarithmic correction term is controversial. Our result shows that the correction to the entropy from the GUP gives an opposite contribution to the area entropy.

Recently, starting with a modified entropy-area relation, Cai, Cao and Hu [13] have shown that the first law of thermodynamics on the apparent horizon can produce a modified Friedmann equation. Now we give the main results of Cai, Cao and Hu's approach and apply their approach to the case of the modified entropy-area relation (22). Suppose the apparent horizon has an entropy $S_G(A)$. Applying the first law of thermodynamics to the apparent horizon of FRW universe, we can obtain the

corresponding Friedmann equations

$$(\dot{H} - \frac{k}{a})S_G'(A) = -\pi(\rho + p), \qquad (23)$$

$$\frac{8\pi G}{3}\rho = -\frac{\pi}{G}\int S_G'(A)(\frac{4G}{A})^2 dA, \qquad (24)$$

where a prime stands for the derivative with respect to A. Eq.(23) and (24) are nothing but the modified first and second Friedmann equation corresponding to the modified apparent horizon entropy $S_G(A)$.

Noticing that $S_G = A_G/4G$ and considering Eq.(16), we can obtain

$$S_G'(A) = \frac{f_G(A)}{4G}. \qquad (25)$$

Substituting (20) and (25) into the modified Friedmann equation (23) and (24), we can obtain the modified Friedmann equations after considering the GUP, that is

$$(\dot{H} - \frac{k}{a})[1 + \pi\alpha^2 l_p^2 \frac{1}{A} + 2(\pi\alpha^2 l_p^2)^2 \frac{1}{A^2}$$
$$+ \sum_{d=3} c_d(4\pi\alpha^2 l_p^2)^{2d}\frac{1}{A^d}] = -4\pi G(\rho + p), \qquad (26)$$

$$\frac{8\pi G}{3}\rho = 4\pi[\frac{1}{A} + \frac{1}{2}\alpha^2 l_p^2 \frac{1}{A^2} + \frac{2}{3}(\pi\alpha^2 l_p^2)^2\frac{1}{A^3}$$
$$+ \sum_{d=3}\frac{c_d}{d+1}(4\pi\alpha^2 l_p^2)^{2d}\frac{1}{A^{d+1}}]. \qquad (27)$$

B. $n > 3$ and n is an odd number

When n is an odd number, substituting (19) into (16) and integrating, we have

$$A_G = A + \sum_{d=1}^{d=\frac{n-3}{2}} c_d(\alpha l_p)^{2d}\frac{n-1}{n-2d-1}A(\frac{n\Omega_n}{A})^{\frac{2d}{n-1}}$$
$$+ c_{\frac{n-1}{2}}(\alpha l_p)^{n-1}n\Omega_n ln A$$
$$+ \sum_{d=\frac{n+1}{2}} c_d(\alpha l_p)^{2d}\frac{n-1}{n-2d-1}A(\frac{n\Omega_n}{A})^{\frac{2d}{n-1}}. \qquad (28)$$

By making use of Bekenstein-Hawking area law, $S = A/4G$, we can obtain the expression of the entropy of the apparent horizon after taking into account the effect of GUP. That is, the correction to entropy is given by

$$S_G = \frac{A}{4G} + \sum_{d=1}^{d=\frac{n-3}{2}} c_d(\alpha l_p)^{2d}\frac{n-1}{n-2d-1}\frac{A}{4G}(\frac{n\Omega_n}{A})^{\frac{2d}{n-1}}$$
$$+ c_{\frac{n-1}{2}}\frac{(\alpha l_p)^{n-1}}{4G}n\Omega_n ln\frac{A}{4G}$$
$$+ \sum_{d=\frac{n+1}{2}} c_d(\alpha l_p)^{2d}\frac{n-1}{n-2d-1}\frac{A}{4G}(\frac{n\Omega_n}{A})^{\frac{2d}{n-1}}$$
$$+ const. \qquad (29)$$

It is obvious that the logarithmic correction term exists when n is an odd number.

In order to obtain the modified Friedmann equations from the modified entropy-area relation (29) for $(n+1)$-dimensional FRW spacetime, we have to generalize Cai, Cao and Hu's approach to a $(n+1)$-dimensional FRW universe, while the original approach in [13] is only valid in $(3+1)$-dimensional FRW universe. The generalization is simple. The first law of thermodynamics on the apparent horizon $dE = TdS$ leads to

$$A(\rho + p)H\tilde{r}_A dt = \frac{1}{2\pi\tilde{r}_A}dS_G, \qquad (30)$$

here $A(\rho+p)H\tilde{r}_A dt = dE$ is the amount of energy having crossed the apparent horizon. By way of some simple manipulations, we can obtain the Friedmann equations in $(n+1)$-dimensional FRW universe, that is

$$(\dot{H} - \frac{k}{a^2})f_G(A) = -\frac{8\pi G}{n-1}(\rho + p), \qquad (31)$$

$$\frac{8\pi G}{n}\rho = -\int f_G(A)(\frac{A}{n\Omega_n})^{\frac{-2}{n-1}}\frac{dA}{A}, \qquad (32)$$

Substituting (19) into (31) and (32), we can obtain the modified Friedmann equations in $(n+1)$-dimensional FRW spacetime including the consideration of the GUP,

$$(\dot{H} - \frac{k}{a^2})[1 + \frac{\alpha^2 l_p^2}{4}(\frac{n\Omega_n}{A})^{\frac{2}{n-1}} + \frac{(\alpha^2 l_p^2)^2}{8}(\frac{n\Omega_n}{A})^{\frac{4}{n-1}}$$
$$+ \sum_{d=3} c_d(\alpha l_p)^{2d}(\frac{n\Omega_n}{A})^{\frac{2d}{n-1}}] = -\frac{8\pi G}{n-1}(\rho + p), \quad (33)$$

$$\frac{16\pi G}{n(n-1)}\rho = (\frac{n\Omega_n}{A})^{\frac{2}{n-1}}$$
$$+ \sum_{d=1}\frac{c_d}{d+1}(\alpha l_p)^{2d}(\frac{n\Omega_n}{A})^{\frac{2d+2}{n-1}}. \quad (34)$$

We note here that the above equations are independent on whether n is an odd or even number. When we take $n = 3$, Eq.(33) and (34) reduce to Eq.(26) and (27) respectively.

C. $n > 3$ and n is an even number

When n is an even number, following the same route above, we can obtain the expression of the entropy of the apparent horizon after taking into account the effect of GUP, which is

$$S_G = \frac{A}{4G} + \frac{\alpha^2 l_p^2}{4}\frac{n-1}{n-3}\frac{A}{4G}(\frac{n\Omega_n}{A})^{\frac{2}{n-1}}$$
$$+ \sum_{d=2} c_d(\alpha l_p)^{2d}\frac{n-1}{n-2d-1}\frac{A}{4G}(\frac{n\Omega_n}{A})^{\frac{2d}{n-1}}. \quad (35)$$

From this expression, when d is an even number, the logarithmic term does not exist in the correction to the

entropy of the apparent horizon of FRW spacetime. This implies that the logarithmic correction term in the entropy of the apparent horizon is dimensional dependent.

Since the derivation of the modified Friedmann equations (33,34) in $(n+1)$-dimensional FRW universe is not relevant to that whether n is an even or odd number, (33,34) are also valid when n is an even. Therefore, the modified Friedmann equations from the modified entropy (35) are just Eqs.(33,34).

III. THE EUP CASE

The GUP is the high energy correction to the conventional Heisenberg uncertainty relation. In large length scales, the GUP is unimportant. In this case, one might consider an extension of the uncertainty relation which contains the effect of the large length scales. The extended uncertainty principle is given by[20, 23]

$$\delta x \delta p \geq 1 + \beta^2 \frac{\delta x^2}{l^2}, \tag{36}$$

where β is a dimensionless real constant, and l is an unknown fundamental characteristic large length scale. The EUP implies that there is a minimal momentum

$$\delta p \geq \frac{1}{\delta x} + \frac{\beta^2}{l^2} \delta x \geq 2|\frac{\beta}{l}|. \tag{37}$$

From EUP (36), the uncertainty of momentum can be written as

$$\delta p \geq \frac{1}{\delta x} + \frac{\beta^2}{l^2} \delta x = \frac{1}{\delta x} f_E(\delta x), \tag{38}$$

where

$$f_E(\delta x) = 1 + \frac{\beta^2 \delta x^2}{l^2} \tag{39}$$

is the departure function in EUP case.

Now, with the same approach in Section II, it is easy to obtain the entropy of the apparent horizon after taking into account the effect of EUP,

$$S_E = \frac{A}{4G} + \frac{4\beta^2}{l^2} \frac{n-1}{n+1} (\frac{A}{n\Omega_n})^{\frac{2}{n-1}} \frac{A}{4G}. \tag{40}$$

The corresponding modified Friedmann equations are expressed as

$$(\dot{H} - \frac{k}{a^2})(1 + \frac{4\beta^2}{l^2}(\frac{A}{n\Omega_n})^{\frac{2}{n-1}}) = -\frac{8\pi G}{n-1}(\rho + p),$$
$$\frac{16\pi G}{n(n-1)}\rho = (\frac{n\Omega_n}{A})^{\frac{2}{n-1}} + \frac{4\beta^2}{l^2} \ln A. \tag{41}$$

When $n = 3$, the modified Friedmann equations are

$$(\dot{H} - \frac{k}{a^2})(1 + \frac{\beta^2}{\pi l^2}A) = -4\pi G(\rho + p),$$
$$\frac{8\pi G}{3}\rho = \frac{4\pi}{A} + \frac{4\beta^2}{l^2} \ln A. \tag{42}$$

Here are some remarks: First, in the derivation of the modified entropy (40), we didn't impose any limit on the inequality (38) as in (19). This means that the entropy (40) is an exact expression. Second, although the derivation of entropy (40) is in the context of the FRW universe, it is very easy to generalize it to the black hole physics. For example, with the similar procedure, one can easily find that the entropy (40) is also valid in Schwarzschild black hole after the consideration of the effect of EUP.

Third, it is of interest to consider a more general case that combine both the GUP and the EUP and is named the generalized extended uncertainty principle (GEUP). The GEUP is given by[20, 24]

$$\delta x \delta p \geq 1 + \alpha^2 l_p^2 \delta p^2 + \beta^2 \frac{\delta x^2}{l^2}. \tag{43}$$

From this expression, it is easy to obtain the uncertainty of momentum

$$\delta p \geq \frac{1}{\delta x} f_{GE}(\delta x^2), \tag{44}$$

where

$$f_{GE}(\delta x^2) = \frac{\delta x^2}{2\alpha^2 l_p^2}[1 - \sqrt{1 - \frac{4\alpha^2 l_p^2}{\delta x^2}[1 + \beta^2 \frac{\delta x^2}{l^2}]}] \tag{45}$$

is the departure function in the case of the GEUP. By closely following the procedure in Section II, one can also obtain a modified entropy of the apparent horizon of the FRW universe and the corresponding modified Friedmann equations including the effect of the GEUP.

IV. SUMMARY AND DISCUSSIONS

In this paper, we have investigated the influence of the GUP and the EUP on the thermodynamics of the FRW universe. We have shown that the GUP and EUP contribute corrections to the conventional entropy-area relation on the apparent horizon of the FRW universe as well as the Friedmann equations. The later implies that the GUP and EUP can influent the dynamics of the FRW universe. In particular, in the case of the GUP, we have shown that the leading logarithmic correction term exists only for the (odd number+1) dimensional FRW spacetime, and moreover, the leading logarithmic term gives a positive contribution to the entropy of the apparent horizon. For (even number+1) dimensional FRW spacetime, there is not a logarithmic correction term in the entropy of the apparent horizon.

It is worthwhile to point out that the results in this paper can be generalized in some ways. First, it is of interest to search the statistics meaning of the modified entropy on the apparent horizon of the FRW universe. In [12], using the brick wall method, the authors have calculated the statistics entropy of a scalar field in FRW universe. How can one modify their results if one take

into account of the effect of the GUP or EUP? This is an interesting problem and it needs further investigation.

Second, we have known that the GUP and EUP can modify the dynamics of the universe. In the early universe, the high energy effects may be important. This means the GUP may play an important role in the early time of our universe. On the other hand, as a large length scale effect, the EUP may be important in the late time universe. Does the modification of the GUP or the EUP to the Friedmann equations have some observational effects? How can one probe them? These problems will be considered in our further workings.

Third, the GUP and the EUP modify the thermodynamics of both black holes and FRW universe. More gen-

erally, it is of great interest to investigate the influence of the GUP or EUP on the Einstein equation. The investigation in this direction may provide a deeper insight into the understanding of the quantum gravity or large length scale corrections to the classical Einstein gravity.

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